Blending Perspectives: Studies of Discourse and Gesturing in Math and Special Education

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Thank You

- To the MSU Math Education Colloquium organizing committee, and especially Gail Burrill and Freda Cruél who helped coordinate our ‘visit’.
- To everyone who is able to join us today.

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Plan for Our Presentation

- A little bit of Anna’s research trajectory, and a little bit of Casey’s.
- A conversation about how our paths came together, and the projects we have worked together on.

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ANNA

Anna’s Background

- A math education researcher
  - Interested in how students participate in mathematical reasoning and problem solving.
  - I primarily conduct discourse analysis, using techniques from systemic functional linguistics (SFL) that I learned from my advisor and others.
  - I also draw upon work in educational psychology and learning sciences.
- I try, as much as possible, to work in the intersection of the interpersonal and mathematical dimensions of doing math, especially doing math in collaboration.
  - Questions of what gets said, who says it, and how it is received by others.
  - Questions of how to measure the contributions of student talk in math, especially over time.

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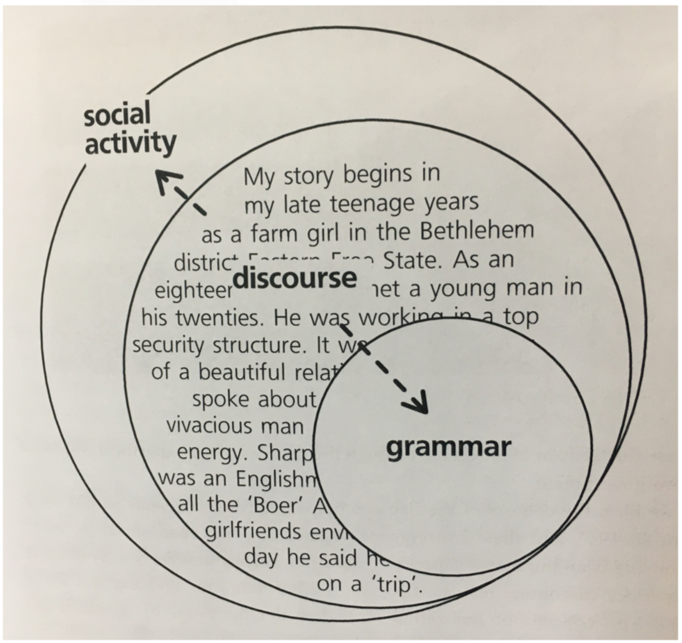
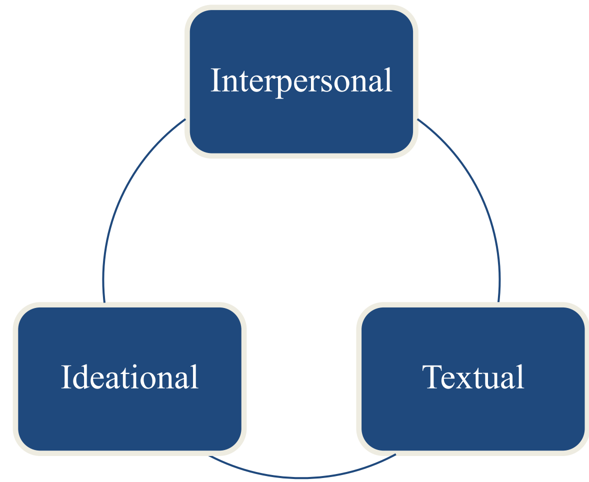
Using SFL to Study Student Discourse



Discourse connecting social activity to grammar (Martin & Rose, 2007, p. 5)



Metafunctions of language, corresponding to social functions we use language for.

Teacher-Student Interactions: Using the System of Negotiation

- Most often, teachers pose questions for which they have an answer in mind (i.e., “I” questions in the IRE sequence).
- “True” questions grant more agency to students but are more difficult for a teacher to manage.
- Initiation questions can serve an important form of “social scaffolding” when students are stuck.

T: So what do you know about isosceles triangles? (*dK1)*

S: That they equal 180? (*K2)*

T: Yeah. *(K1)*

T: Do you all agree with each other? *(K2)*

S: Yeah. *(K1)*

T: And what is it that you all agree on? *(K2)*

S: We agree that if one angle is 45 and we wanted to make a right triangle, then the other one has to be 45. *(K1)*

González and DeJarnette (2012, 2015)
Baxter and William (2010)

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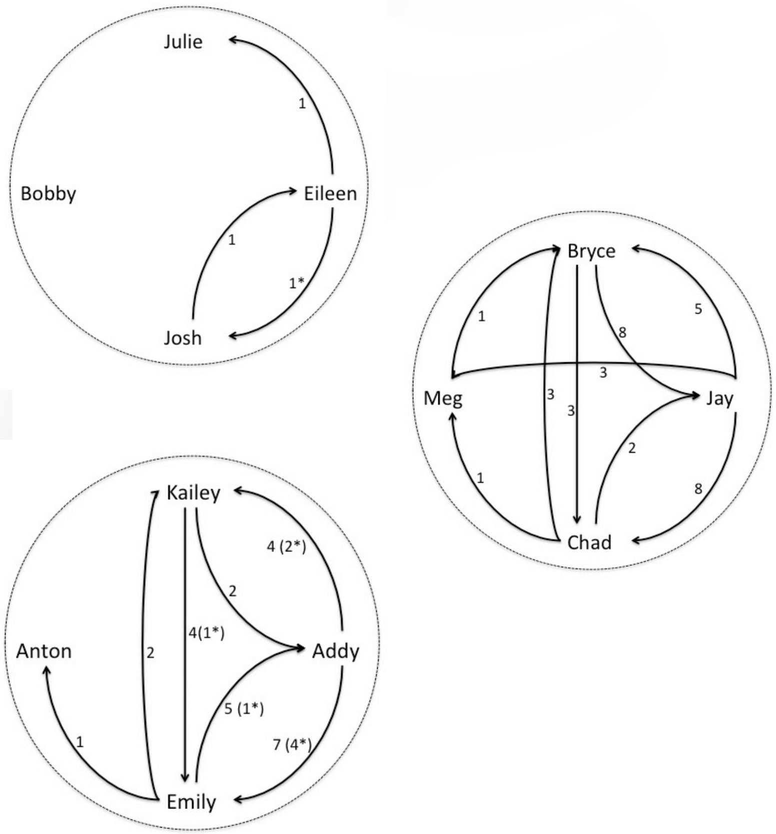
Student Positioning and Mathematical Reasoning



- When students work together in small groups, their patterns of interaction are freer from the constraints of teacher-student positions.
- But students still establish patterns in how they position themselves relative to one another.
- Students who frequently reposition themselves (e.g., taking turns offering information, posing questions, and challenging each other) create more opportunities for mathematical reasoning.

DeJarnette and González (2015)

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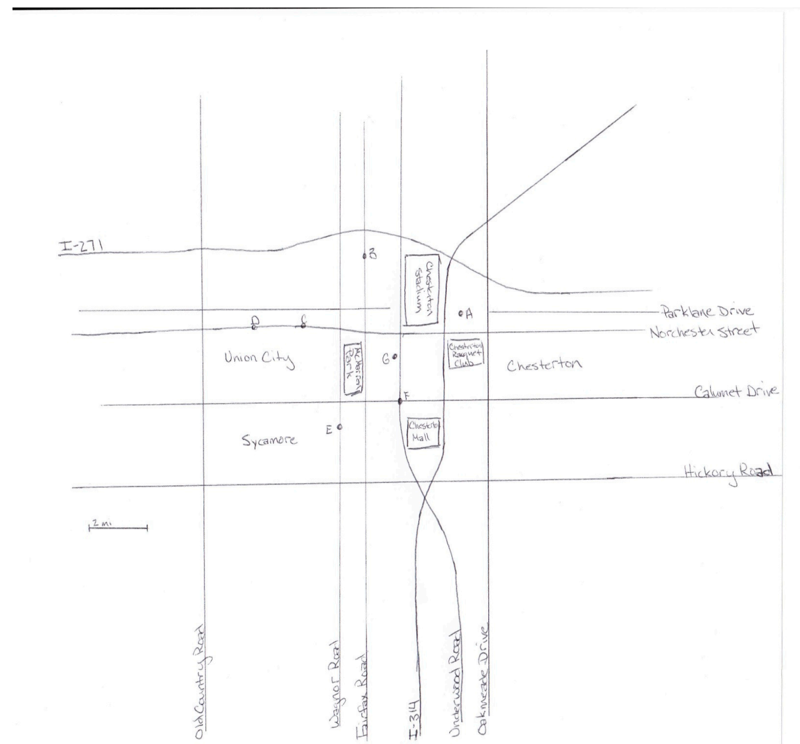
Students Constructing Meaning While Solving Real-World Problems

- In one study we gave students a map of their community, with existing grocery stores marked, and asked them to locate a new store.
- Our goal was to document how students integrated knowledge of mathematics with knowledge of context in their talk.



DeJarnette and González (2016)

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Students Constructing Meaning While Solving Real-World Problems

- Students prioritized their knowledge of the problem context over their knowledge of geometry when solving the problem.
- Students' knowledge of the context informed their mathematical meaning making.
- Students used the same words to construct different mathematical meanings.

DeJarnette and González (2016)

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The Situated Nature of Students' Discourse

- Analysis of how students construct meaning can be difficult, because students most often introduce ideas through presuming references that assume some shared knowledge.
- Students also often reference themselves as actors doing the work.

Megan: Could ***these two figures*** be measured using ***the same driving script***?

Jade: No because ***you*** would have to do ***the opposite***.

Megan: Would ***that*** even work?

Colton: There ***you*** go. [***You***] see? Do ***you*** see where ***it*** starts like that?

Seth: So ***we***'re almost there.

DeJarnette (2016)

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Challenges of Students' Talk About Math

- Interpreting students' mathematical talk requires knowledge of their referents (i.e., what prior knowledge do they share, what shared artifacts are they using).
- The use of informal language gives students space for "trying out the possibilities of words they have encountered" (Barwell, 2005, p. 124).
- However, references to symbols and diagrams create new sets of challenges, because these representations have their own discipline-specific conventions and are used to abstract information.

DeJarnette (2016, 2019)
O'Halloran (2015)
Barwell (2005)

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Overarching Takeaways

- 1-1 tutoring could allow more space for PSTs to make sense of students' referents and build on students' thinking.
- But, doing so would require breaking out of traditional questioning sequences.
- Many of the challenges I encountered in my earlier work foreshadowed the opportunities that Casey and I have tried to create.

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CASEY

Brief Background

- I'm a special education researcher…
  - Who does qualitative research on math interventions for students with mild disabilities
  - Who draws from cognitive psychology, math education, and special education for my research
- An overarching goal of my work is create a sustainable, no-cost model of math tutoring in secondary schools for students with LD that provides…
  - A learning opportunity for pre-service teachers
  - High-quality tutoring for students with LD
  - Extra help for local schools

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Working Memory & Students with LD

- Working memory
  - processing,
  - storing, and
  - integration of information
- Often a problem for students with LD
  - Worse in situations when students are unfamiliar with the concept
  - Worse in multi-step situations

Baddeley (2003)
Barrouillet et al. (2007)
Swanson & Beebe-Frankenberger (2004)

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Offloading Information into Diagrams

- Offloading – storing information on paper rather than in short term memory
  - During read aloud, hear the first part. Write it down.
  - Hear the second part. Write it down
  - Hear the third part. Write it down
  - Then, take a deep breath and think about how all of the parts connect.

Xin (2008)

There are 5 apples in each basket. Stan has 3 baskets. How many apples does he have in total?

$$\boxed{5} \times \boxed{3} = \boxed{a}$$

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Connecting to LTM to support WM in strategic ways

- Connecting to long-term memory supports working memory (i.e., students can think better critically about new and challenging things when we connect to what they already know).
  1. Create a network of knowledge in LTM about a concept using manipulatives (e.g., building things with blocks) or using their experiences outside of school (e.g., talking about the structure of buildings made with Legos or things built in Minecraft) (Concrete)
  2. Draw things or put things into 2-dimensional models (Semi-concrete)
  3. Eventually, use diagrams and then just equations to talk about the concept (Abstract)

Hord & Xin (2015)

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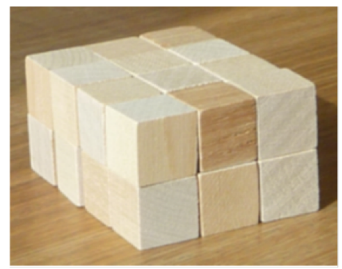
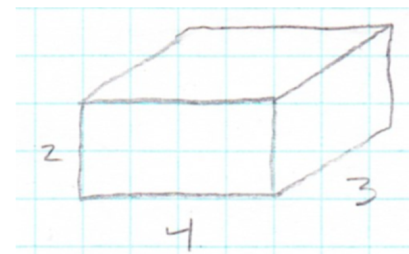
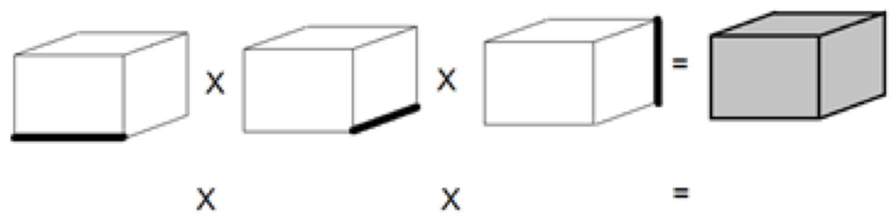
Example of CSA and Diagrams

- What is the volume of a rectangular prism with these dimensions:
  - Length = 2
  - Width = 3
  - Height = 4



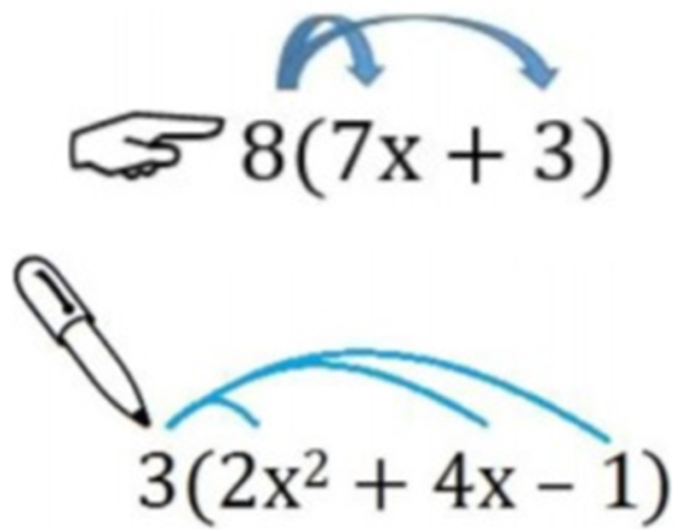




Hord & Xin (2015)

Eventually Though, in Algebra Things Change…

- The curriculum is more abstract
- The students are better at using math notation and the tutors rely on gestures and math notation more and a little less on diagrams and manipulatives.
- I was surprised to find that gestures + math notation on scratch paper matter more than work with younger students.



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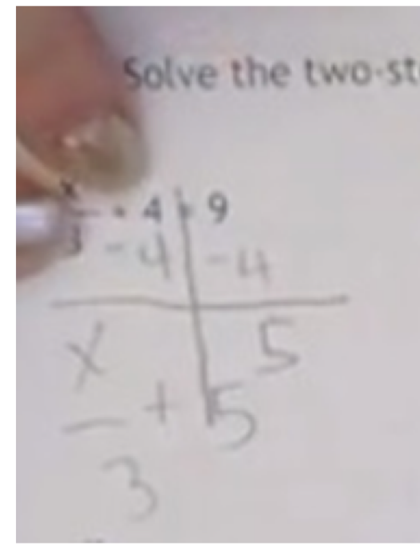
Hord et al. (2020)

Gestures support Working Memory

- Students learn better when…
  - Teachers gesture &
  - Students gesture
- This matters when it involves more demonstrative gestures (e.g., arching motions in distribution) and even just pointing gestures.
- Pointing orients the student
- Then, we can pull back and ask good questions.



Goldin-Meadow & Alibali (2013)
Hord et al. (2016)

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Good Use of Gestures, but Better Questioning Needed

- Tutors were using gestures well, but not necessarily asking good enough questions

- Our next steps for improvement:
  - Good questioning
    - How do we support students in ways that keep them moving forward?
    - How do we still leave time and space for students to think critically?
  - Improving pre-service teachers' knowledge of algebra

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OUR WORK TOGETHER

Our Tutoring Context

- We facilitate partnerships with schools in which pre-service teachers (PSTs) provide 1-1 tutoring for students with identified needs, usually learning disabilities.
- Early in our work, these partnerships were ad-hoc and any type of training for tutors happened on the fly.
- PSTs typically work with students once per week, with an effort to maintain consistent tutor-student pairs.

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Our Current Project

- We recruit five PSTs each year to participate in professional development training and to tutor students in a local school weekly.
- We meet with tutors at the school to develop their knowledge of:
  - Math content
  - The use of gestures and visual representations
  - Questioning practices
- Currently data collection is paused, and we're hopeful to recruit a third cohort next fall.

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The Challenge of Teacher Questioning (from a review of literature)

- PSTs tend to pose primarily lower-order questions, and posing questions may be more challenging in math compared to other subjects.
- Developing content knowledge and noticing skills can support PSTs' questioning practices, especially with respect to posing more specific probing questions.

DeJarnette, Wilke, & Hord (2020)
Diaz et al. (2013)
Kaya & Cevic (2017)
Kilic (2018)
Moyer & Milewicz (2002)
Weiland et al (2014)

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The Challenge of Questioning for PSTs

- When comparing PSTs to more experienced teachers, PSTs do not necessarily have clear rationales for posing lower-order questions like some more experienced teachers do.

DeJarnette, Wilke, & Hord (2020)

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The Case of Tanisha

- Tanisha was an eighth-grade student at a local public school, taking Algebra 1.
- She was selected by her teacher to receive tutoring because, although she was struggling in the course, she was receptive to help, was always talkative, and had good attendance.
- We had audio records and field notes of several tutoring sessions with Tanisha.

DeJarnette, Marita, & Hord (2020)

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Tanisha, Her Tutors, and Slope

1. What connections did Tanisha and her tutors establish among ideas related to slope when working on tasks about linear functions?
2. How did the use of symbolic fraction notation shape the spoken interactions between Tanisha and her tutors?

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Tanisha, Her Tutors, and Slope

- A ratio represented as *a*/*b* can be meaningfully interpreted either as a comparison of two distinct quantities or as a single value (Lobato, Ellis, & Zbiek, 2010).
- In this case, Tanisha used the "comparison of two distinct quantities" construction, while her tutors used the "single value" construction. Their shared written representations obscured the discrepancy.

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Turn #	Speaker	Turn	Semantic Relationships
25	Emily	Now do you think you can find the slope?	
26	Tanisha	Cause, cause, oh yes, I can!	
27	Emily	Perfect.	
28	**Tanisha**	***Y* minus *y*, minus, wait, *y* one. So, *y* sub one, *y* sub two, *x* sub one minus *x* sub two [writing y_1-y_2 and x_1-x_2]. So I'll do 2178 minus 2360. [Tanisha calculates 2178-2360.]**	**Process**
29-34		**[Tanisha and Emily talk through calculations and Tanisha records the value -182/-28 for slope.]**	
35	Emily	So now what should you do to make that a single number?	Quantifier ("single number"
36	Tanisha	But that's my slope.	Quantifier ("that", i.e., 182, 28)
37	Emily	Yes it is. But what can you do to make it easier on yourself?	
38	Tanisha	I could make it a decimal?	Quantifier ("decimal")
39	Emily	Yeah.	

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Tutor Questioning Now

- In our current project, we recruited four Special Education PSTs for the first cohort.
- We spent 15 weeks meeting with the PSTs on a weekly basis, at the school where they provided tutoring.
- PSTs tutored students weekly in Algebra 1 during the semester of training and into the following school year.

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Summary of Topics of the Tutor Training

Sessions 1-4	Linear functions, the use of gestures to support students' attention, theories of learning and how they inform teaching.
Sessions 5-7	Linear functions, funneling and focusing patterns of interaction (Herbel-Eisenmann & Breyfogle, 2005).
Session 8	Solving systems of equations.
Session 9	Systems of equations, "buying time" questions.
Session 10	Reflecting on PSTs' experiences and challenges with students.
Session 11-13	Systems of equations, probing sequences of questions (Franke et al., 2009).
Session 14	Systems of equations, asking versus telling (Baxter & Williams, 2010).
Session 15	Linear functions, multiple representations of a real-world context.

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Questioning Framework

Question Type	Descriptions
Simplest Questions	
Gathering information	Requires immediate, usually short, answer. Elicits discrete pieces of information or steps to solve a problem.
Leading	Similar to gathering information, except this type of question offers the student a set of alternatives to help lead them towards a correct response.
Simpler Questions	
Inserting terminology	Questions that ask specifically for a word, phrase, or notation. This code should be reserved for instances in which the exclusive purpose of the question is to introduce the word or phrase.
Linking	Establishes a connection to a prior task or shared experience.
Connecting to context	Makes a connection to a real-world context, either context that was included in the task or by introducing a new context to make a connection.
More Complex Questions	
Probing	Requires a student to explain why a particular idea or solution makes sense. This can also include questions asking a student to elaborate or clarify their thinking.
Most Complex Questions	
Exploring mathematical meanings or relationships	Points to underlying mathematical relationships that might not be obvious in the given problem. Makes links between mathematical ideas and representations.
Orienting and focusing	Draws a student's attention to key elements of a given problem. This type of question should help enable a student to get started or make progress on a task.
Extending thinking	Extends the situation under discussion to other situations where similar ideas might be used. For this code, the PST must ask about something beyond what is included in a given task.

Boaler & Brodie (2004)

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A Summary of PSTs' Questions

- We transcribed and coded three sessions for each PST.
- All PSTs increased the overall percentage of questions they posed and decreased their percentage of gathering information and leading questions.
- In three cases, the PSTs maintained this trajectory through the follow-up data collection (session 3).
- Overall, PSTs posed more frequent probing questions, and (to a lesser extent) more frequent exploring questions.

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Students' Responses to More Complex Questions

	Total Number Posed	Correct Short Answer	Correct Explanation	Incorrect or Incomplete	No response	PST Left No Time for Response
Probing	41	2 (5%)	16 (39%)	15 (37%)	7 (17%)	1 (2%)
Exploring	36	9 (25%)	7 (19%)	14 (39%)	4 (11%)	2 (6%)
Orienting	22	1 (5%)	1 (5%)	14 (64%)	4 (18%)	2 (9%)

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PSTs' Next Moves When Students Provided Incomplete or Incorrect Responses

	Posed the Same Type or More Complex Question	Posed a Less Complex Question	Corrected the Student or Told Information	Other
Probing	3 (20%)	7 (47%)	4 (27%)	1 (7%)
Exploring	5 (36%)	4 (29%)	5 (36%)	0
Orienting	1 (7%)	7 (50%)	6 (43%)	0

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Summary and Implications

- Students' level of engagement with higher-complexity questions adds to existing literature documenting the wealth of resources that students with LD have for making sense of mathematics in a variety of ways.
- We have to hold two things in our head: Complex questions are, by definition, hard for students to answer. And, all students have knowledge and resources to engage these questions.

Hunt & Empson (2015)
Lambert (2015)

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Summary and Implications

- PSTs can learn to pose higher-complexity questions, especially when they learn about questioning in coordination with developing their content knowledge.
- We need to give PSTs more experience thinking about what might come after a question.

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Tutor Gesturing (Our Current Work)

- Research on gesturing has documented the different forms of gestures (e.g., pointing, sweeping motions).
- We are now analyzing data to document the different functions of PSTs' gestures (e.g., orienting, probing, exploring) in coordination with their questions.
- For example, we are seeing tutors use gestures to orient students to important information and then asking challenging questions to push the students to think critically.

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Tutor Perspectives

- Qualitative analysis of the perceptions of school personnel and pre-service teachers about our program
- The school personnel indicated there was a mutually beneficial relationship between the tutors and the school.
- The perceptions of the tutors revealed challenges they face
  - remembering Algebra I content,
  - posing strategic questions to students,
  - dealing with students' math anxiety, and
  - conveying Algebra I content accessibly.
- The tutors reported positive experiences in the program including learning from field experience.

Hord & DeJarnette (2020)

Conclusions

- Our collaborative work tries to accomplish two overarching goals.
  - Contribute to an ongoing movement to bridge gaps between mathematics education and special education.
  - Implement pedagogies of enactment, so that PSTs can practice specific skills of teaching math with a community of peers and mentors.

Foegen & Dougherty (2017)
Grossman & McDonald (2008)

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QUESTIONS?

A PST Shifting to a Lower-Complexity Question

Turn #	Speaker	Turn	Code
93	Linda	Oh okay. So, five is half of 10, she, so how would you find after 20 days?	Orienting
94	Mia	Five divided by 20?	Incorrect
95	Linda	So she gets candy half the time.	
96	Mia	I am going to continue this out [drawing a diagram].	
97	Linda	Wait, just think about it, she gets candy just half the time. So five is half of the time. Five over 10 is half so what is half of 20?	Leading
98	Mia	10.	Correct Short Answer

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A PST Posing a Sequence of Exploring Questions

Turn #	Speaker	Turn	Code
20	Alice	So how do you know which line is steeper, based on the slope?	Exploring
21	Carly	You could do rise over run.	Incorrect Short Answer
22	Alice	Rise over run. Okay, and what does the rise over run tell you?	Exploring
23	Carly	The slope of the line.	Incomplete Short Answer
24	Alice	The slope. So, once you get the slope how do you know then? You have two lines, one of the slopes was 2/3 and one of the slopes was 1, what is going to be steeper? Which one, the smaller or the bigger?	Exploring/Leading (2) (No Time for Response)
25	Carly	One.	Correct Short Answer

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A Probing Question Followed by Gathering Questions

Turn #	Speaker	Turn	Code
62	Brittany	Which equation could we use to figure out each of these?	Other
63	Bill	[Points to the correct linear function, $y = 450 – 4x$.]	
64	Brittany	Good, why?	Probing
65	Bill	Subtracting.	Incomplete
66	Brittany	Subtracting what?	Gathering
67	Bill	4.	Incomplete
68	Brittany	4?	Other
69	Bill	From 450.	Incomplete
70	Brittany	And what?	Gathering
71	Bill	Multiplying.	Incomplete
72	Brittany	Multiplying what?	Gathering
73	Bill	By x.	Incomplete
74	Brittany	What does x stand for? Do you know?	Connecting to Context
75	Bill	Multiplication. x stands for the amount of candy, no the amount of days.	Explanation

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